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**Volatility Forecasts for the Mexican Peso - U.S. Dollar  
Exchange Rate: An Empirical Analysis of GARCH,  
Option Implied and Composite Forecast Models**

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## Volatility Forecasts for the Mexican Peso - U.S. Dollar Exchange Rate: An Empirical Analysis of GARCH, Option Implied and Composite Forecast Models<sup>\*</sup>

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### Abstract

The volatility accuracy of several volatility forecast models is examined for the case of daily spot returns for the Mexican peso - US Dollar exchange rate. The models applied are univariate GARCH, a multi-variate GARCH (BEKK model), option implied volatilities, and a composite forecast model. The results show that the composite volatility forecasts are superior to the other models in terms of mean squared errors. Conclusions are as follows: the composite model is superior and both type of data -historical and implied in option prices must be used when available.

**Keywords:** Composite forecast models, Exchange rates, Multivariate GARCH, Option implied volatility, Volatility forecasting.

**JEL Classification:** C22, C52, C53, G10

### Resumen

En el presente trabajo de investigación se analiza el poder predictivo de varios modelos de pronósticos de volatilidad diaria del tipo de cambio Peso Mexicano - Dólar Estadounidense. Los modelos que se utilizan son: univariado GARCH; multi-variado GARCH (modelo BEKK); volatilidad implícita de opciones; y, un modelo compuesto. El modelo compuesto fue el más certero al compararlo con el resto de los modelos en términos del error cuadrático medio. Las conclusiones son: el modelo compuesto fue superior al pronosticar y se deben de utilizar ambos tipos de datos -series históricas y de volatilidad implícita de opciones- en especial si estos últimos están disponibles.

**Palabras Clave:** Modelos de pronósticos compuestos, Multivariado GARCH, Pronósticos de volatilidad, Tipo de cambio peso - dólar, Volatilidad implícita de opciones.

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## I. INTRODUCTION

There are basically four general methods widely used to forecast financial volatility of financial variables. These are: 1) by using historical data (price returns), 2) by applying Autoregressive Conditional Heteroscedasticity - type models (ARCH-type), 3) by calculating option implied volatilities (when option data is available); and, 4) by using stochastic volatility models (Poon and Granger: 2003).<sup>1</sup> By extrapolating the estimates of the different models it is possible to obtain volatility forecasts. Even though all of these are widely used by academics and practitioners, nowadays there is a current debate about which method is superior in terms of forecasting accuracy (Brooks: 2002; Poon and Granger: 2003; Andersen et al.: 2005).

The present paper addresses the existing debate in the academic literature related to volatility forecasting accuracy by testing two of these methods: ARCH-type and implied in options. In addition, a composite forecast specification will be constructed with the volatility forecasts of the aforementioned methods. The main objective is to analyze which forecast model is superior in terms of goodness-of-fit; i.e., by comparing their mean squared errors (MSE). At present, there is no individual method statistically proven to be the most accurate, although most of the literature has found that option implieds are superior (Poon and Granger: 2003). Everyday there is more research published on this topic. For example, by 2003,

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<sup>1</sup> Other methods to forecast financial volatility have been suggested. These are: Nonparametric, neural networks, genetic programming and models based upon time change and duration. However, it has been found that these have relatively less predictive power and the number of publications using these methods is substantially lower (Poon and Granger: 2003).

there were about one hundred working papers published (Andersen et al.: 2005; Poon and Granger: 2003).

The models presented in this study are: 1) a univariate Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model (Bollerslev: 1986), 2) a multivariate GARCH model (Engle and Kroner: 1995); and, 3) a composite forecast model (which includes multivariate GARCH and implied volatility forecasts). These models are applied empirically in order to test the following null hypothesis:

$H_0$ : Composite volatility forecasts models do not contain additional information content of the realized (ex post) volatility.

Different to most works in the literature, this paper includes not only a comparison between ARCH-type and option implied volatility, but also statistical tests to find which model combination provides superior accuracy within a composite framework. Finally, it is worth mentioning that the study is carried out for the Mexican peso – USD exchange rate. Up to now, this exchange rate had not been analyzed with the methodology proposed in this paper.

The layout of this paper is as follows. The literature reviews of the ARCH-type, implied volatilities and composite approaches are presented in Section II. The motivation and contribution of this work are presented in Sections III and IV. The models are explained in detail in Section V. Data information is shown in Section VI. Section VII presents the descriptive statistics. The results are presented in Section VIII. Finally, Section IX concludes. Figures and tables can be found in the Appendix.

## II. ACADEMIC LITERATURE OF VOLATILITY FORECAST MODELS

### *II.1. ARCH-type VOLATILITY MODELS*

Volatility of financial variables is described by Brooks (2002) as simply involving calculation of the variance or standard deviation of financial asset's returns -in the usual statistical way- during a certain historical period (or time frame). This variance or standard deviation may become a volatility forecast for all future periods (Markowitz: 1952). This historical volatility measure was traditionally used as a volatility input variable in option pricing models. However, there is growing evidence that the use of volatility predicted from relatively more sophisticated time series models, for example, ARCH-type models, may give more accurate option valuations (Akgiray: 1989; Chu and Freund: 1996). This is because the latter types of models capture the time-varying behavior commonly observed in volatility of financial data. This further explains the volatility clustering also observed in financial time series data.<sup>2</sup> The method for modeling volatility turns out to be more realistic than simply using a constant volatility estimate as it was normally used in the past (Markowitz: 1952). Nowadays, there is strong empirical evidence that financial volatility is, in fact, time-varying (Mandelbrot: 1963; Fama: 1965; Engle: 1982, 2003).

It is well documented that ARCH-type models can provide accurate estimates of price volatility. Just to mention a few, refer to Engle (1982), Taylor

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<sup>2</sup> Volatility clustering means that the variance of log-prices or returns could be high for an extended period and low for another extended period. For example, the variance or volatility of a financial asset daily returns can be high for two months and low for the following two. This type of behavior reinforces the belief that time series volatility are not independently identically distributed (*i.i.d*).

(1985), Akgiray (1989), Bollerslev et al. (1992), Ng and Pirrong (1994), Susmel and Thompson (1997), Wei and Leuthold (1998), Engle (2000), and Manfredo et al. (2001). However, there is less evidence that ARCH-type models give reliable forecasts for out-of-sample evaluations (Park and Tomek: 1989; Schroeder et al.: 1993; Manfredo et al.: 2001). All of them found that the explanatory power of these out-of-sample forecasts is relatively low. In most cases,  $R^2$  are below 10% (Pong et al.: 2003).<sup>3</sup> Thus, the forecasting ability of these models can be highly questionable considering the relatively poor accuracy performance of these models in out-of-sample estimations.

## *II.2. OPTION IMPLIED VOLATILITY MODELS*

Today it is widely known that implied volatilities from options prices are accurate estimators of the price volatility of their underlying assets (Clements and Hendry: 1998; Fleming: 1998; Blair, Poon and Taylor: 2001; Manfredo et al.: 2001; Martens and Zein: 2002; Neely: 2002; Ederington and Guan: 2002; Giot: 2003). The forward-looking nature of implied volatilities is intuitively appealing and theoretically different from the well-known conditional volatility ARCH-type models estimated using backward-looking time series approaches. Within the academic literature there is evidence that the information content of estimated implied volatilities from options could be superior to those estimated with time series approaches. The aforementioned evidence is supported by Fleming et al. (1995)

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<sup>3</sup> They found that implied volatility forecasts performed at least as well as forecasts from historical models, specifically, Autoregressive Fractional Integrated Moving Average Models (ARFIMA). One and three month time horizons were used.

and Giot (2003) for futures market indexes; Jorion (1995), Xu and Taylor (1995), Neely (2002), and Benavides (2004) for foreign exchange; Christensen and Prabhala (1998), Figlewski (1997), Fleming (1998), Clements and Hendry (1998), Blair, Poon and Taylor (2001), and Martens and Zein (2002) for stocks; Ederington and Guan (2002) for futures options of the S&P 500; and Manfredo et al. (2001) and Benavides (2003) for agricultural commodities.

Nonetheless, not all research papers about option implied volatilities are positive in terms of their accuracy. Several research papers are skeptical about the forecasting accuracy of the aforementioned method (Day and Lewis: 1992, 1993; Figlewski: 1997; Lamoureux and Lastrapes: 1993). The latter types of research papers have found serious inconsistencies when calculating option implied volatilities. They argue mainly about the possibility of incorrect specifications of the option pricing models commonly used. These works have increased the already existing controversy regarding which is the best method or model to use in order to obtain the most accurate volatility forecast estimate. This is because, so far, there are no conclusive answers about which is the most consistent procedure (Manfredo et al.: 2001; Brooks: 2002). It is certain to say that for the out-of-sample volatility forecast evaluation, forecasting price return volatilities has entailed a very difficult task, even for option implied volatilities, given that most of the reported results in the academic literature generally have very low explanatory power; i.e., low  $R^2$ .



### *II.3. COMPOSITE FORECAST MODELS*

Other type of method used to forecast financial volatility is the composite forecast. This is a combination of different forecast models. The purpose of this method is to combine such models, in order to obtain a more accurate forecast estimate. The motivation to use a composite approach is mainly related to forecast errors. It is commonly observed that individual forecast models generally have less than perfectly correlated forecast errors. Each of the models in the composite approach is expected to add significant information to the model as a whole, given the statistical difference in their estimated errors (not being perfectly correlated). Decreasing measurement errors by averaging them with several forecast models could improve forecasting (Makridakis: 1989). It is also said that the variance of post-sample errors can be reduced considerably with composite forecast models (Clemen: 1989).

Composite approaches of financial asset prices started to be formally evaluated since the late 1960s. Among the works on this topic are Bates and Granger (1969), Granger and Ramanathan (1984), Clemen (1989), Makridakis (1989), and Kroner et al., (1994). However, for the volatility forecasting literature these are relatively rare. Blair et al. (2001), Vasilellis and Meade (1996) have done work on stock indexes; and Fang (2002), Pong et al. (2003) and Benavides (2004), on exchange rates. For agricultural commodities, Manfredo et al., (2001) and Benavides (2003).

Bessler and Brandy (1981) created the weights for the composite forecast model based on the forecast ability of each individual model in terms of their MSE.

They found that for quarterly hog prices, the results were superior when these models were combined.<sup>4</sup> Along the same lines, Park and Tomek (1989) evaluated several forecast models (including ARIMA, Vector-Autoregression and OLS for their variances) and concluded in favor of the composite approach. In an opposite finding, Schroeder et al. (1993) reported that forecasting cattle feeding profitability gave conflicting results. Their results show that there was no forecast model consistent enough to consider a reliable forecast model (including the composite model). Manfredo et al. (2001) attempted to forecast agricultural commodity price volatility using several models which included ARIMA, ARCH and implied volatility from options on futures contracts. They found that, based on their MSE, there was no superior model to forecast volatility. However, they recognized that composite approaches, which included GARCH and option implied volatility models performed marginally better than individual forecast models. They also acknowledged that composite approaches could be more widely used when more option data is available. A similar method to that proposed by Manfredo et al. (2001) is applied in the present research paper. Following is the method for an emerging economy exchange rate.

### III. MOTIVATION

The main motivation behind the present research is to contribute to extend the current literature on forecasting financial data volatility. The objective is to compare the predictive accuracy of the most widely used methodologies as never

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<sup>4</sup> Bessler and Brandy analyzed quarterly hog prices for the sample period from 1976:01 to 1979:02.

done before. Special emphasis is given to the composite forecast approach performance versus the accuracy of the models that are not combined. This is because significantly less research about the accuracy of composite forecast models has been done relative to other methods. Up to now, these types of models have not been tested between each other using the Mexican peso – US Dollar (USD) exchange rate, a further motivation to do the proposed research.

#### IV. CONTRIBUTION

The present paper broadens the contribution made in previous research projects related to forecast foreign exchange volatility in several ways. First, several ARCH-type models -which are not commonly applied in the academic literature- are used (specifically, bi-variate and tri-variate models). As it is known, most of the studies in the academic literature apply only univariate models and not the multivariate type. Second, forecast estimates are rigorously compared with each other to evaluate if they are statistically different. Statistically significance tests for equal forecast accuracy have been rarely reported in the literature. This is relevant inasmuch as estimates of these types of models are expected to be statistically different from each other. If this is not the case, then it does not make any difference to use one model or the other. Last, the fact that multivariate GARCH and option implied volatility forecasts are combined in one model is another contribution, given that these specific types of models have not been analyzed within the proposed composite framework.

The empirical analysis of the Mexican peso –USD exchange rate in this research area is new in the economic literature. Most of the literature is based on currencies of developed economies. Individual characteristics of this emerging economy exchange rate like, for example, ‘the peso problem’ can be analyzed by reviewing if the models used here capture some of that unusual behavior in the exchange rate.<sup>5</sup>

The findings of this work could be of interest for agents involved in making risk management decisions related to exchange rates, particularly, the one analyzed in this paper. Such agents could be bankers, policy makers, investors, exchange rate futures traders, central bankers, and academic researchers, among others.

## V. THE MODELS

### *V.1. ARCH-type VOLATILITY MODELS*

The ARCH-type models under analysis are the univariate GARCH( $p, q$ ) and a restricted version of the multi-variate GARCH BEKK( $p, q$ ) model proposed by Engle and Kroner (1995). These models were chosen from the ARCH-family given that they can capture very well the dynamics of exchange rate volatility. For example, ARCH-type models that capture asymmetric volatility (EGARCH, TGARCH, and QGARCH, among others) are not theoretically justifiable for exchange rate volatility modeling. This is because exchange volatilities do not

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<sup>5</sup> In international financial markets ‘the peso problem’ refers to situations where large discrete jumps in exchange rate prices or shifts on policy regimes are observed (Levich: 1998, pp. 237).

exhibit asymmetric volatilities like other financial assets do; i.e., there is no proven statistical evidence that negative volatilities are higher than positive ones for exchange rates. Fractionally integrated ARCH models (FIGARCH  $p, d, q$ ) could be applied for this case but there is an important drawback. For the case of positive  $I(d)$  processes, there is a positive drift or a time trend in volatility. For volatility time trends are usually not observed (Granger: 2001). Thus, theoretically speaking, the GARCH( $p, q$ ) and the BEKK models are consistent to apply in this case.

The BEKK model (named after an earlier working paper by Baba, Engle, Kraft and Kroner (Baba et al.: 1992)) is used in order to estimate the ARCH-type volatilities of the exchange rate under study in a multi-variate framework. The model not only estimates conditional variances, but also conditional covariances. The BEKK model can be useful to test economic theories, which involve price volatility analysis, such as price uncertainty influences to employment variability (Engle and Kroner: 1995). Others could be volatility relationships between financial assets; i.e., CAPM volatility (Bollerslev et al.: 1988), and hedge ratio volatility for stock index returns (Brooks, Hendry and Persaud: 2002), among others.

The univariate GARCH(1,1) model is estimated applying the standard procedure, as explained in Taylor (1986) and Bollerslev (1986). The formulae for the GARCH(1,1) is explained as follows. Two main equations are included in the model: the mean equation and the variance equation.

Mean equation,

$$\Delta y_t = \mu + e_t \quad (1)$$

$$e_t | I_{t-1} \sim N(0, h_t),$$

Variance equation,

$$h_t = \alpha_0 + \alpha_1 e_{t-1}^2 + \beta_1 h_{t-1} . \quad (2)$$

Where  $\Delta y_t$  = first differences in natural logarithms of the exchange rate at time  $t$ ,  $e_t$  is the error term at time  $t$ ,  $I_{t-1}$  is the information set at time  $t-1$ ,  $h_t$  represents the conditional variance at time  $t$  and  $t-1$  for  $h_{t-1}$ . The Greek letters  $\mu$ ,  $\alpha_0$ ,  $\alpha_1$ ,  $\beta_1$  are parameters and  $N(0, h_t)$  is for the assumption that log-returns are normally distributed through time. In other words, assuming a constant mean  $\mu$  (the mean of the series  $y_t$ ) the distribution of  $e_t$  is assumed to be Gaussian with zero mean and variance  $h_t$ . The parameters were estimated using maximum likelihood methodology applying the BHHH (Berndtand, Hall, Hall, and Hausman) algorithm of Berndt et al. (1974). The Bollerslev and Wooldridge (1992) methodology was used to estimate standard errors. The objective log-likelihood function to be maximized is the following:

$$\ln L(\theta) = -\frac{1}{2} \sum_{t=1}^n [\ln(2\pi) + \ln(h_t(\theta)) + z_t^2(\theta)],$$

where  $\theta$  is the set of parameters  $(\mu, \omega, \alpha_i, \beta_i)$  estimated that maximize the objective function  $\ln L(\theta)$ .  $z_t$  represents the standardized residual calculated as  $\frac{\Delta y_t - \mu}{\sqrt{\sigma_t^2}}$ . The rest of the notation is the same as expressed previously.

The procedure to obtain the estimates of the BEKK model is explained as follows, let  $y_t$  be a vector of returns at time  $t$ ,

$$y_t = \mu + \varepsilon_t \quad (3)$$

where  $\mu$  is a constant mean vector and the heteroskedastic errors  $\varepsilon_t$  are multivariate normally distributed,

$$\varepsilon_t | I_{t-1} \sim N(0, H_t) .$$

Each of the elements of  $H_t$  depends on  $q$  lagged values of squares and cross products of  $\varepsilon_t$  as well as on the  $p$  lagged values of  $H_t$ .<sup>6</sup>

Considering a multivariate model setting it is convenient to stack the non-redundant elements of the conditional covariance matrix into a vector; i.e., those elements on and below the main diagonal. The operator, which performs the

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<sup>6</sup> The original dimension of the vector is  $2 \times 1$ . This is because originally there are two series under analysis, exchange rates and interest rates. In any different case it could be extended to a  $n \times 1$  vector. For example, for the tri-variate case, three series are considered. These are: the exchange rate and the interest rates for both economies.

aforementioned stacking, is known as the *vech* operator. Defining  $h_t = \text{vech}(H_t)$  and  $\eta_t = \text{vech}(\varepsilon_t \varepsilon_t')$  the parameterization of the variance matrix is

$$h_t = \alpha_0 + \alpha_1 \eta_{t-1} + \dots + \alpha_q \eta_{t-q} + \beta_1 h_{t-1} + \dots + \beta_p h_{t-p}. \quad (4)$$

Equation 4 above is called the *vech* representation. Bollerslev et al. (1988) have proposed a diagonal matrix representation, in which each element in the variance matrix  $h_{jk,t}$  depends only on their past values and on past values of the cross product  $\varepsilon_{j,t} \varepsilon_{k,t}$ . In other words, the variances depend on their own past squared residuals and the covariances depend on their own past cross products of the relevant residuals. A diagonal structure of the matrices  $\alpha_i$  and  $\beta_i$  is assumed in order to obtain a diagonal model in the *vech* representation shown in Equation 4.

It is difficult to ensure positive definiteness in the estimation procedure of the conditional variance matrix from the above representations. This could estimate negative variances, which is not consistent with statistics theory. To ensure the condition of a positive definite conditional variance matrix in the optimization process, Engle and Kroner (1995) proposed the BEKK model. This model representation can be observed in the following equation:

$$H_t = \omega \omega' + \sum_{i=1}^q \alpha(\varepsilon_{t-i} \varepsilon_{t-i}') \alpha' + \sum_{i=1}^p \beta H_{t-i} \beta' \quad (5)$$



In Equation 5,  $\omega\omega'$  is symmetric and positive definite and the second and third terms in the right-hand-side of this equation are expressed in quadratic forms. This quadratic form ensures that  $H_t$  is positive definite and that no constraints are necessary on the  $\alpha_i$  and  $\beta_i$  parameter matrices. As a result, the eigen values of the variance-covariance matrix have positive real parts, which satisfies the condition for a positive definite matrix that estimates positive variances.

For an empirical implementation, and without loss of generality, the BEKK model can be estimated in a restricted form having  $\omega$  as a 2 x 2 lower triangular matrix,  $\alpha$  and  $\beta$  being 2 x 2 diagonal matrices. Thus, for the bivariate case, the Bi-variate-BEKK model (BVBEKK) can be expressed in the following matrix form:

$$\begin{bmatrix} H_{11,t} & H_{12,t} \\ H_{21,t} & H_{22,t} \end{bmatrix} = \begin{bmatrix} \omega_1 & 0 \\ \omega_2 & \omega_3 \end{bmatrix} \begin{bmatrix} \omega_1 & \omega_2 \\ 0 & \omega_3 \end{bmatrix} + \begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1}^2 & \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} & \varepsilon_{2,t-1}^2 \end{bmatrix} \begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix} \\ + \begin{bmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{bmatrix} \begin{bmatrix} H_{11,t-1} & H_{12,t-1} \\ H_{21,t-1} & H_{22,t-1} \end{bmatrix} \begin{bmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{bmatrix} \quad (6)$$

or (multiplying the matrices),

$$H_{11t} = \omega_1^2 + \alpha_1^2 \varepsilon_{1t-1}^2 + \beta_1^2 H_{11t-1}$$

$$H_{22t} = \omega_2^2 + \omega_3^2 + \alpha_2^2 \varepsilon_{2t-1}^2 + \beta_2^2 H_{22t-1}$$

$$H_{12t} = H_{21t} = \omega_1\omega_2 + \alpha_1\alpha_2 \varepsilon_{1t-1}\varepsilon_{2t-1} + \beta_1\beta_2 H_{12t-1}$$

Following the procedure for the bi-variate case, a tri-variate-BEKK model (TVBEKK) can also be derived. Thus, the specification for the TVBEKK is as follows:

$$H_{11t} = \omega_1^2 + \alpha_1^2 \varepsilon_{1t-1}^2 + \beta_1^2 H_{11t-1} \quad (7)$$

$$H_{22t} = \omega_2^2 + \omega_4^2 + \alpha_2^2 \varepsilon_{2t-1}^2 + \beta_2^2 H_{22t-1}$$

$$H_{33t} = \omega_3^2 + \omega_5^2 + \omega_6^2 + \alpha_3^2 \varepsilon_{3t-1}^2 + \beta_3^2 H_{33t-1}$$

$$H_{12t} = H_{21t} = \omega_1 \omega_2 + \alpha_1 \alpha_2 \varepsilon_{1t-1} \varepsilon_{2t-1} + \beta_1 \beta_2 H_{12t-1}$$

$$H_{13t} = H_{31t} = \omega_1 \omega_3 + \alpha_1 \alpha_3 \varepsilon_{1t-1} \varepsilon_{3t-1} + \beta_1 \beta_3 H_{13t-1}$$

$$H_{23t} = H_{32t} = \omega_2 \omega_3 + \omega_4 \omega_5 + \alpha_2 \alpha_3 \varepsilon_{2t-1} \varepsilon_{3t-1} + \beta_2 \beta_3 H_{23t-1}$$

The variables used in the bi-variate model are the exchange rate ( $y_1$ ) and the Mexican risk-free interest rate ( $y_2$ ). For the tri-variate case, in addition to  $y_1$  and  $y_2$  a new variable is added: the risk-less foreign interest rate ( $y_3$ ). These variables are of relevant use for the theoretical framework of the Uncovered Interest Parity.<sup>7</sup> The specification of these historical ( $p, q$ ) models was selected by applying the Akaike Information Criterion (AIC).<sup>8</sup> The parsimonious first order specification is found to have the smallest AIC. Thus, it is the optimal one compared to the rest.

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<sup>7</sup> This states that the expected change in the exchange rate should be equal to the interest rate differential between that available risk-free in each of the currencies (Brooks: 2002). Algebraically this can be expressed as  $S^e_{t+1} - S_t = (r - r_f)t$ , where  $S$  represents the spot exchange rate,  $S^e_{t+1}$  is the expected spot exchange rate,  $r$  is the domestic risk-less interest rate,  $r_f$  is the foreign risk-less interest rate, and  $t$  expresses that the equation is through time.

<sup>8</sup> The AIC is obtained with the following formula:  $\frac{-2l}{n} + \frac{2k}{n}$ . Where  $l$  is the value of the log likelihood function using the  $k$  estimated parameters,  $k$  is the number of estimated parameters, and  $n$  is the number of observations.

## *V.2. OPTION IMPLIED VOLATILITY*

The option implied volatility of an underlying asset is the market's forecast of the volatility of such asset, obtained from the options written on the underlying asset (Hull: 2003). To calculate the option implied volatility of an asset an option valuation model together with inputs for that model are needed. The inputs for a typical option valuation model are risk-free rate of interest, time to maturity, price of the underlying asset, the exercise price, and the price of the option (Blair, Poon and Taylor: 2001). Using an inappropriate valuation model will produce significantly large pricing errors and option implied volatilities will be mis-measured (Harvey and Whaley: 1992). For each trading day the aforementioned implied volatilities are derived from at-the-money (ATM) over-the-counter (OTC) one month option contracts for the Mexican peso - USD.

## *V.3. THE COMPOSITE FORECAST MODEL*

In the spirit of Makridakis (1989), a composite forecast model is also estimated. The composite forecast model includes estimates of the ARCH-type models as well as estimates from implied volatilities. Considering that the time variable in the option price formula is measured in years, estimates of the implied volatilities are calculated on an annualized basis. In order to be consistent using daily returns, the implied volatilities estimates in the composite forecast model

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must be transformed into daily trading-days estimates and then extended to a desired forecast horizon. Following Manfredo et al. (2001) the formula to transform the aforementioned annualized estimates into daily trading-days implied volatilities, which can be extended to a desired forecast horizon ( $hr$ ), is presented in the following equation:

$$\hat{\sigma}_{t,hr} = IV_t \cdot \frac{\sqrt{hr}}{\sqrt{252}} \quad (11)$$

In Equation 11,  $\hat{\sigma}_{t,hr}$  represent the  $hr$ -period volatility forecast for the exchange rate at time  $t$ . The symbol  $IV_t$  represents the implied volatility estimate (annualized) at time  $t$ . The  $hr$  represents the desired forecast horizon. Considering that daily implied volatilities estimates are obtained on an annualized basis with daily data, the numerator in Equation 11 is one, which represents one-trading-day (in other words, the forecast is made for the next available trading day) and the denominator (number 252) represents the approximate number of trading days in one year.

In order to create the composite forecast model it is necessary to use a simple averaging technique where the composite forecast is merely the average of individual forecasts at time  $t$ . It follows that weights for each of the volatility forecasts are generated by an ordinary least squares (OLS) regression of past realized volatility on the respective volatility forecasts. This procedure to create the weights for the aforementioned composite volatility forecast is explained in more

detail in Granger and Ramanathan (1984). This can be observed in the following equation:

$$\sigma_t = \alpha_0 + \beta_1 \hat{\sigma}_{1,t} + \beta_2 \hat{\sigma}_{2,t} + \dots + \beta_k \hat{\sigma}_{k,t} + \varepsilon_t \quad (12)$$

In Equation 12,  $\sigma_t$  represent the realized volatility at time  $t$ , and  $\hat{\sigma}_{k,t}$  represent the individual volatility forecast ( $k$ ) corresponding to the realized volatility at period  $t$ . As it can be observed in this equation, the composite forecast model includes the average of the individual volatility forecasts at time  $t$ . Following Blair, Poon and Taylor (2001) the realized volatility can be calculated as follows:

$$\sigma_{t,hr}^2 = \sum_{j=1}^{hr} R_{t+j}^2 \quad (13)$$

where  $\sigma_{t,hr}$  represents the realized (ex-post) volatility at time  $t$  over forecast horizon  $hr$ . The  $R_t^2$  represents the squared log return at time period  $t$ . It is important to point out that the volatility is not observed. The realized volatility represents a 'proxy' for the real volatility.<sup>9</sup> However, this method is the most commonly used in the volatility forecasting literature (Andersen and Bollerslev: 1998; Poon and Granger: 2003; Andersen, et al. 2005). Thus, the resulting composite volatility forecast can be observed in Equation 14 where the variables are the same as expressed previously,

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<sup>9</sup> I am thankful to Daniel Chiquiar and Carlos Capistrán for asking me to clarify this.

$$\hat{\sigma}_{t+1} = \hat{\alpha}_0 + \hat{\beta}_1 \hat{\sigma}_{1,t+1} + \hat{\beta}_2 \hat{\sigma}_{2,t+1} + \dots + \hat{\beta}_k \hat{\sigma}_{k,t+1} . \quad (14)$$

The composite forecast model of this equation is a one-day volatility forecast. In order to create a composite volatility forecast of more than one trading day; i.e.,  $hr > 1$ , the estimated one-day composite volatility forecast (from Equation 14) is multiplied by  $\sqrt{hr}$ . The aforementioned method for obtaining a composite volatility forecast of more than one day ( $h > 1$ ) is a common practice in the academic literature; however, it is important to emphasize that an alternative is to obtain predictions of volatility for each period in the forecast interval (e.g. from an ARCH model).

The MSE obtained from each of the estimates of all volatility forecast models are compared with each other. The formula to obtain the MSE is presented in Equation 15,

$$MSE = \frac{1}{n} \sum_{i=1}^n \left( \sigma_{t,hr,i}^2 - \hat{\sigma}_{t-1,hr,i} \right)^2 , \quad (15)$$

where  $n$  is equal to the number of observations and the other variables are the same as described previously. These MSE comparisons are performed in order to provide a robust analysis of the accuracy of the aforementioned composite volatility forecast model against the alternative models (the conditional and implied volatilities models). The model with the smaller MSE is considered the most accurate volatility forecasting model of the returns of the exchange rate. Ranking

models in terms of their MSE is a common practice in forecasting volatility literature (Manfredo et al.: 2001). The procedure applied to obtain these statistical significances is based on the method postulated by Diebold and Mariano (1995).<sup>10</sup>

## VI. DATA

### VI.1. OPTIONS AND SPOT DATA

The data for the spot exchange rate Mexican peso-USD consists of daily spot prices obtained from Banco de México's web page database.<sup>11</sup> These are daily averages of quotes offered by major Mexican banks and other financial intermediaries. The option implied data is calculated from daily OTC options for 1-month to maturity contracts of the Mexican peso-USD (the time to maturity of the option contract is always fixed and equal to one month). The data was downloaded from Bloomberg database. The ticker is USDMXNV1M.<sup>12</sup> The data for the interest rates consists of daily 30-day interest rates of Mexican Certificates of Deposit (CDs) obtained from Banco de México's web page. US CDs were obtained from

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<sup>10</sup> This method requires generating a time series, which is the differential of the squared-forecast errors from two different forecast models; i.e.,  $d_t = (\sigma_t^2 - \hat{\sigma}_{1,t-1})^2 - (\sigma_t^2 - \hat{\sigma}_{2,t-1})^2$ , where  $d_t$  is the differential of the series and  $\hat{\sigma}_i$  is the forecast of the  $i$  model. The  $t$ -statistic is obtained in the following way,  $\frac{\bar{d}}{\sqrt{\frac{sd}{n}}}$  where  $\bar{d}$  is the sample mean and  $sd$  is equal to the standard deviation of the

series ( $d$ ). The notation for the other variables is the same as described previously.

<sup>11</sup> Banco de México's Web page is <http://www.banxico.org.mx>

<sup>12</sup> Option implied volatility data obtained from a well-known international financial institution was also used. However, the results in the estimations show that the implied volatilities obtained from Bloomberg quotes were more accurate in terms of statistical tests. It was therefore decided to include only the latter for the present analysis. I am thankful to Alejandro Díaz de León for encouraging me to analyze additional series.

the Federal Reserve (FED) web page with the same maturity.<sup>13</sup> The sample period under analysis is more than five years, from 01/03/2000 to 01/09/2006. The sample size consists of 1,295 daily observations.

## VII. DESCRIPTIVE STATISTICS

This subsection presents the descriptive statistics for the ex post realized volatilities of exchange rate returns and volatility forecasting models. Prior to fitting the GARCH models, ARCH effects tests were conducted on the series under analysis in order to corroborate if the series had ARCH effects, and therefore assure that these types of models are appropriate for the data. The test conducted was the ARCH-LM test following the procedure of Engle (1982). According to the results, all the series under study; i.e., the spot and the interest rates, had ARCH effects.<sup>14</sup> Under the null of homoscedasticity in the errors, the *F*-statistics were 8.5119 for the spot, 12.9234 for the Mexican interest rates and 24.6407 for the US interest rates. For all variables the null hypotheses were rejected in favor of heteroscedasticity on those errors. Thus, the application of ARCH models is statistically justified (the critical value is 6.63 at the limit for a 1% confidence level).

**Figure 1** presents the spot exchange rate Mexican pesos per USD and its realized volatility. **Table 1** shows the descriptive statistics for the realized volatility and the forecasting models. As it can be observed in Table 1, the means of the

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<sup>13</sup> The FED web page is <http://www.federalreserve.gov/>

<sup>14</sup> These tests were conducted by regressing the logarithmic returns of the analyzed series under analysis against a constant. The ARCH-LM test is performed on the residuals of that regression. The test consists on regressing the square residuals against a constant and lagged values of the same square residuals. The statistical significance is tested with a *F*-Distribution. Five lags were applied in each test.



option implied volatilities are those with higher values. These findings are consistent with Christensen and Prabhala (1998), who found that option implied volatilities had higher first moments. The distributions of all variables are highly skewed and leptokurtic, indicating the non-normality of the daily returns and the forecast estimates. This is consistent with the work of Wei and Leuthold (1998), who also corroborated the existence of non-normality in this type of data frequency. Last, **Figures 2 and 3** presents the observations of the realized volatility (top line) and estimates of the GARCH models and option implieds (bottom lines). It can be observed that in both graphs all models capture the volatility clustering periods shown with the realized volatility. At a simple sight, the implied volatility models estimates are significantly greater than the realized volatility (Figure 3).

## VIII. RESULTS

### *VIII.1. IN-SAMPLE EVALUATION*

The MSE results are presented in **Table 2**. For the composite model the BVBEKK and the option implied were chosen given that they had superior forecast accuracy relative to their counterparts. The weights assigned to each model were obtained from an OLS regression as explained in Section V.3. above. The option implied volatilities had the higher weight, which was nearly 90%. On the other hand, the BVBEKK model obtained only 10% of the weight. This shows that option implied volatilities had higher information content compared with the multi-variate GARCH model. From Table 2 it can be observed that the most accurate model is

the composite model given that it has the lowest MSE.<sup>15</sup> The second best forecast is the option implied volatility. When tests for statistical difference between the two competing models were applied, the null hypothesis of equal forecasting accuracy was rejected (see Table 3). This leads to the conclusion that there is forecast superiority between the composite model and its counterparts. These results are consistent with part of the literature that favors composite models in terms of better forecasting accuracy. MSE differences among models (Table 3) are statistically significant at 1%.

## *VIII.2. OUT-OF-SAMPLE EVALUATION*

The sample period under analysis is partitioned in half in order to evaluate the out-of-sample forecasts. Estimates (in-sample) for all models are obtained from January 3<sup>rd</sup>, 2000 to January 22<sup>nd</sup>, 2003 for a total of 647 observations (about half the total number of observations). The jump-off period is January 23<sup>rd</sup>, 2003. Thus, the out-of-the-sample evaluation for all forecasting models is from January 23<sup>rd</sup>, 2003 to January 9<sup>th</sup>, 2006.

The forecast models chosen for the composite specification were those with superior forecast accuracy (lowest MSE) in the in-sample evaluation. These were the BVBEKK for the ARCH-type models and the option implieds. The weights applied for the forecast estimates were qualitatively similar to those used in the in-sample valuation i.e. around 90% for the option implied volatilities and around 10% for the BVBEKK. The results of the MSE for each model including the composite

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<sup>15</sup> It is important to point out that several sample periods were tested for the composite model. These results were qualitatively similar to the ones presented in the present subsection. These results are available upon request.

specification are presented in **Table 4**. As observed in Table 4, in the out-of-the-sample evaluation the composite model also has the lowest MSE. The second best was option implied volatilities. The MSE statistical differences are presented in **Table 5**. The results are qualitatively similar to those obtained for the in-sample estimates. The forecasts were also estimated using rolling window and a recursive approach. Since the results on both methods were qualitatively similar to the ones explained above they are not reported.<sup>16</sup>

### *VIII.3. ANALYSIS OF THE RESULTS*

The overall finding is that the composite model was superior in terms of MSE. In statistical terms, the composite model's estimated forecasts are statistically different than its counterparts. Thus, the null hypothesis presented that composite volatility forecasts models do not contain additional information content of the realized (ex post) volatility is rejected. The predictive power of option implied volatilities has proven to be more accurate than the ARCH-type models. But if the ARCH-type and option implied forecasts are combined in a composite approach, the MSE becomes lower and statistically significant. This recommends the use of both types of data when available. Finally, the results of this paper are in line with those studies which favor composite specifications (Vasilellis and Meade: 1996; Blair et. al.: 2001; Manfredo et. al.: 2001; Benavides: 2003, 2004). Also, that option implied volatilities have more information content compared to ARCH-type models. About 76% of similar types of studies have found that options implied volatilities are more accurate in forecasting financial variables volatility compared with ARCH-

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<sup>16</sup> These results are available upon the reader's request.

type models (Poon and Granger: 2003). The results of the present paper are consistent with these studies.

## IX. CONCLUSION

The on-going debate regarding which is the most accurate model to forecast volatility of price returns of financial assets has led to a substantial amount of research. Many have compared ARCH-type models against option implied volatilities and composite forecast models. Albeit the majority of the literature advocates the use of option implied volatilities as the most accurate alternative to forecast price returns volatilities, no conclusion has been drawn in terms of finding one superior model. This is because the statistical evaluation of the forecasts has generally shown that the competing models have statistically equal accuracy.

In the present research paper the aforementioned volatility forecast models; i.e., ARCH-type, option implieds and composite forecast models, were compared with each other to find the most accurate volatility forecasting model for the daily spot returns of the Mexican peso – US Dollar exchange rate. Tests were performed for both in-sample and out-of-sample evaluations. Rolling windows and recursive methods were also applied. Even though option implied volatilities contained most of the information content of the realized spot return volatility, the composite forecast was superior one. The weights in the composite specification were obtained with a OLS regression. These were about 90% for option implied volatilities and about 10% for the bi-variate BEKK model. In addition, there was statistical significant difference between ARCH-type models, option implieds and composite approach forecasts. The null hypothesis of no additional information

content for composite models is, therefore, rejected. Considering the evaluation of forecast estimates in terms of their statistical differences, it is concluded that the composite model is the most accurate. Finally, a word of caution is given to these conclusions given that not all assumptions were met when estimating the models. Specially, the normality assumption for the ARCH-type models is highly questioned.

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## APPENDIX

Figure 1. Exchange rate Mexican peso – USD in levels (right axis) and its realized volatility (left axis).

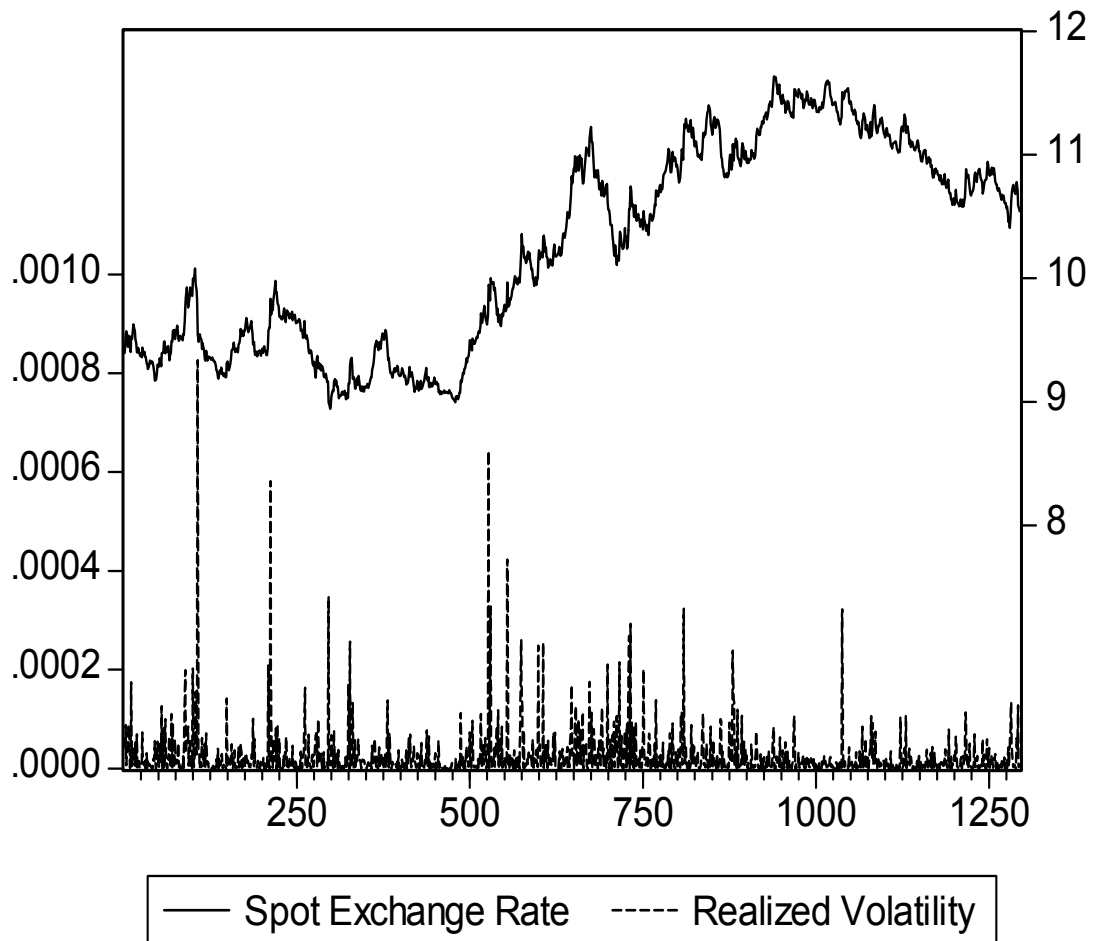


Table 1. Descriptive statistics for realized (ex post) volatility and volatility forecasting models estimates for the daily returns Mexican Peso-USD spot exchange rate

Model	Mean	Variance	Skewness	Kurtosis	N
Realized volatility	$2.61 \times 10^{-5}$	$2.87 \times 10^{-9}$	6.6501	71.2398	1,294
GARCH(1,1)	$2.67 \times 10^{-5}$	$1.66 \times 10^{-10}$	2.7950	15.3874	1,294
BVBEKK(1,1)	$2.62 \times 10^{-5}$	$7.59 \times 10^{-11}$	1.9312	8.5458	1,294
TVBEKK(1,1)	$2.60 \times 10^{-5}$	$2.84 \times 10^{-11}$	0.2379	2.0209	1,294
Option implied	$3.38 \times 10^{-5}$	$2.13 \times 10^{-10}$	0.5361	2.3210	1,294
Composite forecast	$6.96 \times 10^{-6}$	$2.00 \times 10^{-9}$	5.1928	46.1972	1,294

This table reports the descriptive statistics of the realized (ex post) volatility and the volatility forecasting models estimates for the Mexican peso-USD exchange rate. Option implied data corresponds to at-the-money (or near-the-money) options and it was supplied by Bloomberg. The ticker is USDMXNV1M. The realized (ex post) volatility used to obtain the composite forecast model is the annualized ex post daily spot squared return. The sample size is 1,294 observations (one observation was lost due to the lags in the models), from January 3<sup>rd</sup>, 2000 to January 9<sup>th</sup>, 2006. *N* = Number of observations.

Figure 2. Realized (ex post) volatility and volatility estimates from ARCH-type models (realized volatility on the top part of the graph).

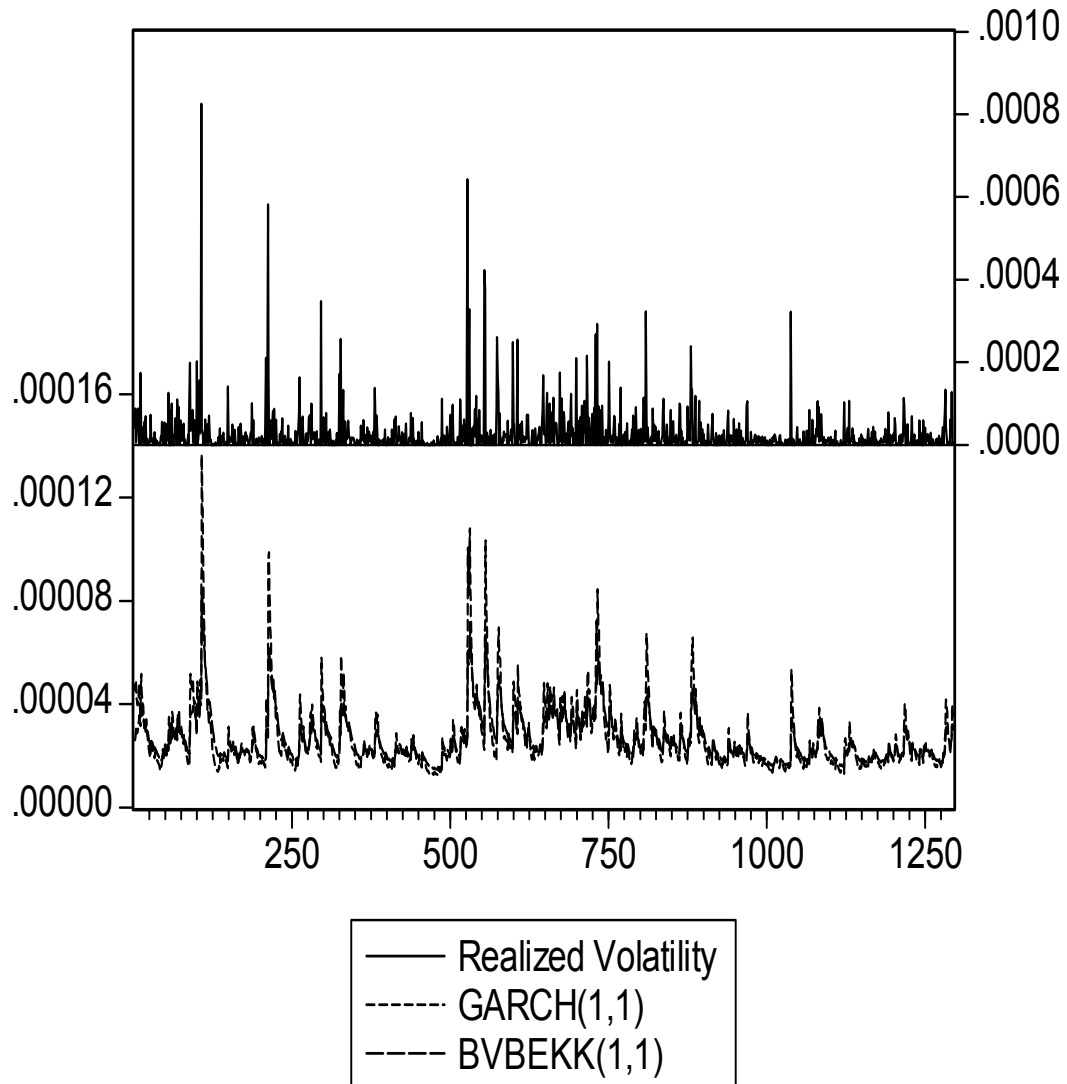


Figure 3. Realized (ex post) volatility and volatility estimates from option implied models (realized volatility on the top part of the graph)

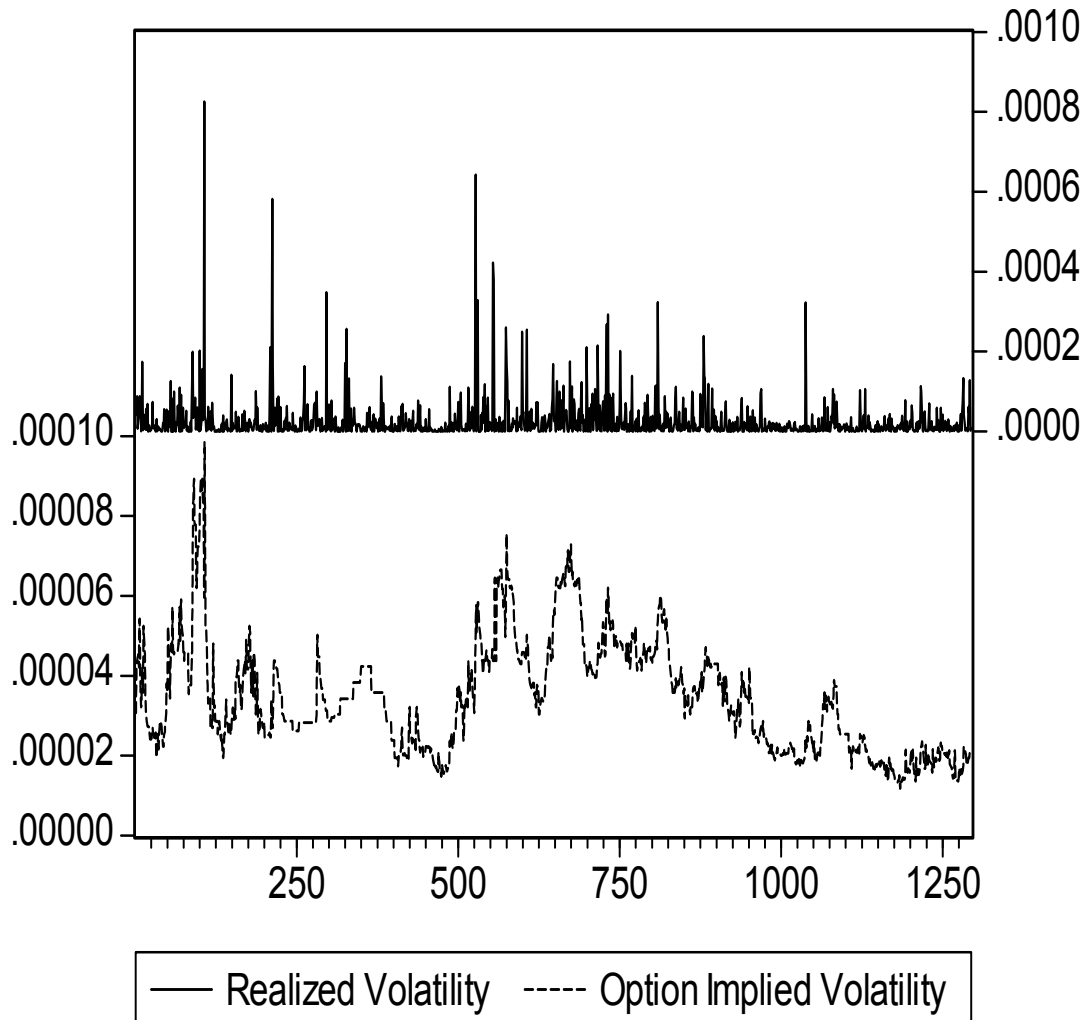


Table 2. In-sample MSE for the Mexican peso – USD spot exchange rate forecasts

FORECAST MODEL	MSE IN-SAMPLE	RANK
Composite model	<b>2.7262 x 10<sup>-9*</sup></b>	<b>1</b>
Option implieds	2.7904 x 10 <sup>-9</sup>	2
BVBEKK(1,1)	2.8008 x 10 <sup>-9</sup>	3
GARCH(1,1)	2.8142 x 10 <sup>-9</sup>	4
TVBEKK(1,1)	2.8245 x 10 <sup>-9</sup>	5

This table reports the Mean-Square-Error (MSE) of the volatility forecasting models for the daily spot returns for the Mexican peso - USD exchange rate. Results are for in-sample forecasts. Option implied data corresponds to at-the-money (or near-the-money) options and it was supplied by Bloomberg. The ticker is USDMXNV1M. The realized (ex post) volatility used to obtain the MSE is the annualized ex-post daily spot return volatility for the sample period under analysis. Rank 1 represents highest, rank 5 represents lowest. The sample size is 1,294 observations (one observation was lost due to the lags in the models), from January 3<sup>rd</sup>, 2000 to January 9<sup>th</sup>, 2006. Bold (\*) indicates the smallest value.

Table 3. Statistical difference between volatility forecasts for in-sample evaluation.

	Composite	Option Implieds	BVBEEK	GARCH	TVBEEK
Composite	N.A.	27.5092*** (0.0000)	22.5655*** (0.0000)	22.4878*** (0.0000)	22.5625*** (0.3104)
Option implieds		N.A.	17.6781*** (0.0000)	14.5526*** (0.0000)	20.1515*** (0.0000)
BVBEEK			N.A.	0.9539 (0.4020)	1.0146 (0.3104)
GARCH				N.A.	1.8083* (0.0707)
TVBEEK					N.A.

This table reports statistical significantly differences in MSE for the in-sample forecasts. Each model in the rows is tested against each model in the columns. The method used is the one postulated by Diebold and Mariano (1995).  $t$ -statistics reported.  $p$ -value in brackets.  $H_0$  = forecasts are statistically equal.  $t$ -statistic (\*\*\*) Indicates the coefficient is statistically significant at 1% confidence level; (\*\*) Indicates the coefficient is statistically significant at 5% confidence level; (\*) indicates the coefficient is statistically significant at 10% confidence level. The sample size is 1,294 observations (one observation was lost due to the lags in the models), from January 3<sup>rd</sup>, 2000 to January 9<sup>th</sup>, 2006. N. A. = Not applicable.



Table 4. Out-of-sample MSE for Mexican peso – USD spot exchange rate forecasts

FORECAST MODEL	MSE IN-SAMPLE	RANK
Composite model	<b>1.6589 x 10<sup>-5*</sup></b>	<b>1</b>
Option implieds	1.7009 x 10 <sup>-5</sup>	2
BVBEKK(1,1)	1.7583 x 10 <sup>-5</sup>	3
GARCH(1,1)	1.8634 x 10 <sup>-5</sup>	4
TVBEKK(1,1)	1.8947 x 10 <sup>-5</sup>	5

This table reports the Mean-Square-Error (MSE) of the volatility forecasting models for the daily spot returns for the Mexican peso - USD exchange rate. Results are for out-of-sample forecasts. Option implied data correspond to at-the-money (or near-the-money) options and it was supplied by Bloomberg. The ticker is USDMXNV1M. The realized (ex post) volatility used to obtain the MSE is the annualized ex-post daily spot return volatility for the sample period under analysis. Rank 1 represents highest, rank 5 represents lowest. The sample size is 1,294 observations (one observation was lost due to the lags in the models), from January 3<sup>rd</sup>, 2000 to January 9<sup>th</sup>, 2006. Bold (\*) indicates the smallest value.

TABLE 5. Statistical difference between volatility forecasts for the out-of-sample evaluation.

	Composite	Option Implieds	BVBEEK	GARCH	TVBEEK
Composite	N.A.	19.1532*** (0.0000)	11.3182*** (0.0000)	9.2582*** (0.0000)	9.3675*** (0.3104)
Option implieds		N.A.	9.7268*** (0.0000)	5.2489*** (0.0000)	10.0258*** (0.0000)
BVBEEK			N.A.	0.0011 (0.9991)	0.0008 (0.994)
GARCH				N.A.	0.0002 (0.9998)
TVBEEK					N.A.

This table reports statistical significantly differences in MSE for out-of-sample forecasts. Each model in the rows is tested against each model in the columns. The method used is the one postulated by Diebold and Mariano (1995).  $t$ -statistics reported.  $p$ -value in brackets.  $H_0$  = forecasts are statistically equal.  $t$ -statistic (\*\*\*) Indicates the coefficient is statistically significant at 1% confidence level; (\*\*) Indicates the coefficient is statistically significant at 5% confidence level; (\*) indicates the coefficient is statistically significant at 10% confidence level. The sample size is 1,294 observations (one observation was lost due to the lags in the models), from January 3<sup>rd</sup>, 2000 to January 9<sup>th</sup>, 2006. N. A. = Not applicable.